Reg. No. | | | | | | | | |

Fifth Semester B.Sc.3 Degree Examination, November/December 2017
(Regular and Repeaters w.ef 2016-17 New Syllabus)
MATHEMATICS (Optional)
Paper – III : Dynamics and Calculus of Variations

Time : 3 Hours Max. Marks : 80

Instructions : *1) Question paper has **3** Parts namely **A, B** and **C**.*
*2) Answer **all** questions.*

PART – A

1. Answer **any ten** of the following : **(10×2=20)**

a) For the particle moving along the curve $r = e^{\theta}$ prove that the radial velocity is equal to the transverse velocity.

b) Write the expressions for the tangential and normal accelerations.

c) The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr^2 and $\mu\theta^2$. Show that the equation to the path is

$$\frac{\lambda}{\theta} = \frac{\mu}{2r^2} + C.$$

d) Prove that the length of the perpendicular from the pole to the tangent is equal to the radius vector at an apse.

e) Define apsidal angle and apsidal distance.

f) Prove that the maximum horizontal range is at an angle of projection 45°.

g) A particle is projected from a point on a level ground with velocity of 98 m/sec at an angle of elevation of 30°. Find the time of flight.

h) State Newton's experimental law on direct impact.

i) Define functional and give an example.

j) Show that $f - y'\frac{\partial f}{\partial y'}$ = constant, when f is independent of x.

k) What are isoperimetric problems ?

l) State "Brachistochrone problem".

PART – B

Answer **any four** of the following : **(4×5=20)**

2. Find the tangential and normal velocities along a plane curve of moving particle.

3. A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration towards the origin is always zero show that the transverse acceleration varies as the fifth power of $\operatorname{cosec}\theta$.

4. Find the differential equation of the path in polar form.

5. Find the loss of kinetic energy due to direct impact.

6. Show that the general solution of the Euler's equation for the integral

   $\int_a^b \frac{1}{y}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ is $(x - h)^2 + y^2 = k^2$.

7. Find the extremals of the isoperimetric problem $I[y(x)] = \int_{x_0}^{x_1} y'^2\,dx$ given that

   $\int_{x_0}^{x_1} y\,dx = c$, a constant.

PART – C

Answer **any four** of the following : **(4×10=40)**

8. a) Derive an expression for radial and transverse accelerations of a moving particle in a plane curve.

   b) A point moves in a plane curve so that its tangential acceleration is constant and the magnitudes of the tangential velocity and normal accelerations are in a constant ratio. Find the intrinsic equation of the curve.

9. a) Derive the expression for velocity of the particle at any point of central orbit in the form $v^2 = h^2\left[\left(\frac{du}{d\theta}\right)^2 + u^2\right]$ and also show that $v \propto \frac{1}{P}$.

   b) A particle describes the curve $r^n = A \cos n\theta + B \sin n\theta$ under a force 'f' to the pole. Find the law of force.

10. a) Find the equation of path of projectile.

    b) A ball is projected so as just clear two walls the first of height 'a' at a distance 'b' from the point of projection and second of height 'b' at a distance 'a' from the point of projection. Find the horizontal range.

11. a) State and prove Euler's equation.

    b) Find the extremal of the functional $I[y(x)] = \int_0^{\pi}\left[y'^2 - y^2 + 4y\cos x\right]dx$, $y(0) = 0$, $y(\pi) = 0$.

12. a) Define geodesic and show that the shortest distance between two points in a plane is a straight line.

    b) Prove that the extremal of the isoperimetric problem $I[y(x)] = \int_1^4 y'^2 dx$ with $y(1) = 3$, $y(4) = 24$ subjected to the condition $\int_1^4 y\, dx = 36$ is a parabola.

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