Reg. No.

Fifth Semester B.Sc. 3 Degree Examination, Nov./Dec. 2016
MATHEMATICS (Optional)
Paper – I : Real Analysis
(Regular w.e.f. 2016-17) (Fresh New Syllabus)

Time : 3 Hours | Max. Marks : 80

Instructions : *1) Question paper has **3** Parts namely **A, B** and **C**.*
*2) Answer **all** questions.*

PART – A

1. Answer **any ten** of the following : **(10×2=20)**

a) Define upper and lower Riemann integrals.

b) Give an example of a function which is bounded but not R-integrable.

c) State first mean value theorem of integral calculus.

d) Prove that $\frac{\pi}{4} \le \int_0^{\pi/4} \sec x \, dx \le \frac{\pi}{2\sqrt{2}}$.

e) State Abel's test for the convergence of an improper integral.

f) Discuss the convergence $\int_1^{\infty} \frac{dx}{x^{1/2}(1+x)^{1/3}}$.

g) Test the convergence of $\int_0^{\pi/2} \frac{dx}{\sqrt{\tan x}}$.

h) Evaluate $\int_0^1 x^8 (1-x)^7 dx$.

i) Prove that $\int_0^{\infty} e^{-x^2} . dx = \frac{\sqrt{\pi}}{2}$

j) Evaluate $\int_0^{\pi/2} \sin^5 \theta . d\theta$ by Beta-Gamma function.

k) Evaluate $\int_{0}^{1}\int_{1}^{2} x^3y^3 dx\, dy$.

l) Evaluate $\int_{0}^{1}\int_{0}^{1}\int_{0}^{1} (x+y+z) dx\ dy\ dz$.

PART – B

Answer **any four** of the following : **(4×5=20)**

2. If f(x) and g(x) are bounded and R-integrable in [a, b] then prove that f(x) + g(x) is bounded and R-integrable in [a, b].

3. If f(x) is bounded and R-integrable in [a, b] and M, m are bounds of f(x) then prove that $m(b-a) \leq \int_{a}^{b} f(x)\, dx \leq M(b-a)$.

4. If f(x) and g(x) are positive in [a, ∞) and $\lim_{x\to\infty} \frac{f(x)}{g(x)} = l$ (non zero and finite) then prove that the integrals $\int_{a}^{\infty} f(x)dx$ and $\int_{a}^{\infty} g(x)dx$ behave alike, where a > 0.

5. Test the convergence of $\int_{0}^{\infty} \frac{x^{a-1}}{(1+x)} dx$.

6. Define Gamma function and discuss its convergence.

7. Evaluate $\iint_R xy\,(x+y)\ dx\ dy$ over the region R between $y = x^2$ and y = x.

PART – C

Answer **any four** of the following : **(4×10=40)**

8. a) State and prove condition of R-integrability.

   b) If f is defined on [0, a], a > 0 by $f(x) = x^3\ \forall x \in [0, a]$ then prove that $f \in R[0, a]$ and $\int_{0}^{a} f(x)dx = \frac{a^4}{4}$.

9. a) State and prove fundamental theorem of integral calculus.

b) Prove that $\frac{\pi^2}{9} \leq \int_{\pi/6}^{\pi/2} \frac{x}{\sin x} dx \leq \frac{2\pi^2}{9}$.

10. a) State and prove Dirichlet's test for the convergence of an improper integral of product of two functions.

b) Prove that $\int_0^\infty \frac{\sin kx}{x} dx$ converges.

11. a) Prove that $\beta(m, n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma(m+n)}$ where $m > 0, n > 0$.

b) Prove that $\int_0^\infty x^2 e^{-x^4} dx \cdot \int_0^\infty e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$.

12. a) State and prove Leibnit'z theorem for differentiation under integral sign.

b) Prove that $\int_0^{\pi/2} \frac{\log(1+\cos\alpha \cdot \cos x)}{\cos x} dx = \frac{\pi^2}{8} - \frac{\alpha^2}{2}$.
