15531/E 644

Reg. No. | | | | | | | | |

V Semester B.A./B.Sc. Degree Examination, November 2015

Mathematics (Optional)

(KUD – Repeaters)

Paper – III : STATICS AND LAPLACE TRANSFORMS

Time : 3 Hours] [Max. Marks : 80

Instruction : *Answer* ***All*** *questions.*

I. Answer any **five** of the following : **(5 × 2 = 10)**

1. Define couple and moment of a couple.

2. State the conditions of equilibrium of a system of coplanar forces acting at different points of a rigid body.

3. State the converse of Lami's theorem.

4. Define catenary and common catenary.

5. Evaluate : $L\{\cos^2 t\}$.

6. Find the Laplace transform of $2t^2 + 3t - 4$.

7. Evaluate : $L^{-1}\left\{\frac{1}{s^2 + 4s + 20}\right\}$.

8. Define Heaviside unit step function.

II. Answer any **eight** of the following : **(8 × 5 = 40)**

9. Prove that two coplanar couples of equal and opposite moments balance each other.

10. Derive the Cartesian equation of a common catenary.

11. Prove that the algebraic sum of moments of the forces of a couple about any point in its plane is the same.

12. Forces 1, 2, 3, 5, *P*, *Q* act along *AB, BC, CD, DA, AC, BD* of a square *ABCD* respectively. Find the values of *P* and *Q* for the system reduces to a couple.

13. A given length '2*s*' of an uniform chain has to be hung between points at a same level and tension has not to exceed the weight of a length '*b*' of the chain. Show that the greatest span is $\left(\sqrt{b^2 - s^2}\right)\log\left(\frac{b+s}{b-s}\right)$.

14. The moment of a coplanar forces not in equilibrium about three collinear points *A, B, C* in their plane are G_1, G_2, G_3 respectively. Prove that $G_1 \cdot BC + G_2 \cdot CA + G_3 \cdot AB = 0$.

15. Find (a) $L[e^{2t}(2t^2 - 3t + 4)]$; (b) $L[t^2 + 2^2]$.

16. If $L\{f(t)\} = F(s)$ and $g(t) = \begin{cases} f(t-a), & t > a \\ 0, & t < a \end{cases}$. Then prove that $L\{g(t)\} = e^{-as}F(s)$.

17. Find $L\{t\,e^{-2t} \cdot \cos 2t\}$.

18. Using Convolution theorem, find $L^{-1}\left[\frac{1}{s(s^2+1)}\right]$.

19. Derive the formula for $L(t^n)$, when $n > 0$, and integer.

20. Find $L\left(\frac{\sin^2 t}{t}\right)$.

III. Answer any **three** of the following : **(3 × 10 = 30)**

21. (a) State and prove Varignan's theorem on moments.

    (b) Forces $P - Q$, P, $P + Q$ act at a point in directions parallel to the sides of an equilateral triangle taken in order, find the resultant.

22. (a) With usual notations derive the equation of the form $x = c\,\log(\sec\psi + \tan\psi)$.

    (b) A uniform chain of length '*l*' is suspended from points in the same horizontal line. If the tension at the highest point is twice at the lowest point. Show that the span is $\frac{l}{\sqrt{3}}\log[2 + \sqrt{3}]$.

23. (a) Define dirac-delta function. Hence find its Laplace transform.

(b) If $f(t) = t^2$ for $0 < t < 2$ and $f(t+2) = f(t)$ for $t > 2$, find $L[f(t)]$.

24. (a) State and prove convolution theorem.

(b) Verify convolution theorem for $f(t) = t$ and $g(t) = \cos t$.

25. (a) Find Laplace transform of $\int_0^t f(t)\, dt$.

(b) Solve : $9y'' - 6y' + y = 0$ given $y(0) = 3$ and $y'(0) = 1$.
