25538/E 380

Reg. No. | | | | | | | | |

V Semester B.A.2/B.Sc.2 Degree Examination, November 2015

Mathematics (Optional)

(RCU – Regular and Repeaters)

Paper – II : NUMERICAL ANALYSIS

Time : 3 Hours] [Max. Marks : 80

Instruction : *Answer* ***all*** *questions.*

Students are allowed to use Scientific Calculators.

I. Answer **any ten** of the following : **(10 × 2 = 20)**

1. Explain Secant method to find the real root of the equation $f(x) = 0$.

2. Find the real root of $2x = \cos x + 3$ using iteration method in three stages.

3. Solve by Gauss elimination method : $3x - 2y = 5$ and $x + 3y = -2$.

4. Solve by Gauss-Jordan method : $3x + 4y = 7$ and $7x - y = 6$.

5. Prove that $\nabla = E^{-1}\Delta$.

6. Prove that $\Delta \log [f(x)] = \log \left[\frac{1 + \Delta f(x)}{f(x)}\right]$.

7. Evaluate $\Delta^3 (1 + 2x)(1 + 4x)(1 + 6x)$ where $h = 1$.

8. Write the normal equation of the line $y = ax + b$.

9. State Newton-Gregory backward interpolation formula.

10. Write the formula to find the first derivative using forward difference.

11. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal rule by taking $h = 0.5$.

12. State Simpson's $\left(\frac{3}{8}\right)^{\text{th}}$ rule to evaluate $\int_a^b f(x)\, dx$.

II. Answer **any six** of the following : **(6 × 5 = 30)**

13. Find the real root of $x^3 - 9x + 1 = 0$ in the interval (2, 4), corrected to 3 decimal places by Bisection method.

14. Find the real root of the equation $x^3 + x - 1 = 0$ by using fixed point iteration method corrected to four decimal places.

15. Solve by Gauss elimination method :

$2x + 5y + 7z = 52$; $2x + y - z = 0$; $x + y + z = 9$.

16. Solve the system of equations : $10x + y + z = 12$; $x + 10y + z = 12$; $x + y + 10z = 12$ using Gauss-Jordan method.

17. State and prove Newton-Gregory forward interpolation formulae.

18. Express the function $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ and its successive differences in a factorial notations when $h = 1$.

19. Find $f'(1.2)$ and $f''(1.2)$ from the following table.

x :	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y :	2.72	3.32	4.06	4.96	6.05	7.39	9.02

20. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\left(\frac{3}{8}\right)^{\text{th}}$ rule by taking 7 ordinates. Hence obtain the approximate value of π.

III. Answer **any three** full questions : **(3 × 10 = 30)**

21. (a) Derive Newton-Raphson formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

(b) Find the root of the equation $xe^x = \cos x$ using the Secant method, corrected to four decimal places.

22. (a) Explain Gauss-Seidel iteration method to solve the equations :

$a_1x + b_1y + c_1z = d_1$; $a_2x + b_2y + c_2z = d_2$; $a_3x + b_3y + c_3z = d_3$.

(b) Solve by Gauss-Jordan method :

$x_1 + 2x_2 + x_3 = 8$; $2x_1 + 3x_2 + 4x_3 = 20$; $4x_1 + 3x_2 + 2x_3 = 16$.

23. (a) If $f(x)$ is a polynomial of n^{th} degree in x, then prove that $\Delta^n f(x)$ is a constant and $\Delta^{n+1} f(x) = 0$.

(b) With usual notations, prove that

$$U_0 + \frac{U_1}{1!}x + \frac{u_2}{2!}x^2 + \frac{U_3}{3!}x^3 + \cdots = e^x\left[U_0 + \frac{x}{1!}\Delta U_0 + \frac{x^2}{2!}\Delta^2 U_0 + \cdots\right].$$

24. (a) State and prove Lagrange's interpolation formula for unequal intervals.

(b) By the method of least square, fit a straight line $y = ax + b$ for the following data :

x :	0	5	10	15	20
y :	7	11	16	20	26

25. (a) State and prove 'General quadrature formula' for equidistant ordinate.

(b) Use Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule to find $\int_0^{0.6} e^{-x^2}\, dx$ by taking 6 sub intervals.