# 25537/E 370

Reg. No. | | | | | | | | |

**V Semester B.A.2/B.Sc.2 Degree Examination, October/November 2014**

**Mathematics (Optional)**

**(Regular & Repeaters)**

**Paper – I : REAL ANALYSIS**

Time : 3 Hours] [Max. Marks : 80

***Instruction :*** *Answer* ***all*** *questions.*

I. Answer **any ten** of the following : **(10 × 2 = 20)**

1. Define upper and lower Riemann integrals of a bounded function $f(x)$ in $[a, b]$.

2. State Darboux theorem.

3. If $f(x)$ is defined in $[a, b]$ as

   $f(x) = 1$ when $x$ is rational

   $= -1$ when $x$ is irrational.

   Show that $f(x)$ is bounded but not $R$-integrable in $[a, b]$.

4. State Bonnet's mean value theorem.

5. Show that $\left|\int_p^q \frac{\sin x}{x}\,dx\right| \le \frac{2}{p}$ where $q > p > 0$.

6. State Dirichlet's test for the convergence of an improper integral.

7. Show that $\int_0^{\sqrt{2}} \frac{dx}{\sqrt{2 - x^2}}$ is convergent.

8. Examine the convergence of $\int_1^\infty e^{-x^2} dx$.

9. Prove that $\overline{|n+1} = n!$.

10. Prove that $\int_1^\infty \frac{x^8(1-x^6)}{(1+x)^{24}} dx = 0$.

11. Evaluate $\int_0^1 \int_0^2 (x+y)\, dx\, dy$.

12. Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2 yz\, dx\, dy\, dz$.

II. Answer **any six** of the following : **(6 × 5 = 30)**

13. If $f(x)$ is bounded and integrable in $[a, b]$ and $M$, $m$ are bounds of $f(x)$ in $[a, b]$ then prove that $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$.

14. State and prove Fundamental theorem of integral calculus.

15. Prove that every continuous function in $[a, b]$ is R-integrable in $[a, b]$.

16. If $f(x)$ and $g(x)$ are positive in $[a, b]$ and $f(x) \leq g(x)\ \forall\ x \in [a, b]$ then prove that

(a) if $\int_a^b g(x)dx$ is convergent then $\int_a^b f(x)dx$ is also convergent.

(b) if $\int_a^b f(x)dx$ is divergent then $\int_a^b g(x)dx$ is also divergent.

17. Discuss the convergence of

(a) $\int_0^1 \frac{1}{x^2(1+x^2)} dx$

(b) $\int_0^\infty \frac{1}{(1+x)\sqrt{x}} dx$.

18. Derive the relation between Beta and Gamma functions.

19. Prove that $\frac{\overline{|n}}{t^n} = \int_0^\infty x^{n-1} e^{-tx}\, dx$.

20. Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between $y = x^2$, $y = x$.

III. Answer **any three** of the following : **(3 × 10 = 30)**

21. (a) State and prove the necessary and sufficient condition for integrability of a bounded function $f(x)$ in $[a, b]$.

(b) If $f(x) = \frac{1}{3^{r-1}}$ when $\frac{1}{3^r} < x < \frac{1}{3^{r-1}}$

$= 0$ when $x = 0$

Where $r = 1, 2, 3, \cdots$ then

Show that $f(x)$ is R-integrable in $[0, 1]$ and find $\int_0^1 f(x)\, dx$.

22. (a) State and prove first mean value theorem.

(b) Show that $\frac{1}{3\sqrt{2}} \leq \int_0^1 \frac{x^2}{\sqrt{1+x^2}}\, dx \leq \frac{1}{3}$

by using first mean value theorem.

23. (a) State and prove Abel's test for convergence of an improper integral.

(b) Test the convergence of $\int_0^\infty \sin x^2\, dx$.

24. (a) Derive the Duplication formula

$\sqrt{\pi}\, \overline{|2m} = 2^{2m-1} \overline{|m}\, \overline{\left| m + \frac{1}{2} \right.}$.

(b) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$.

25. (a) With usual notations prove that

$$\frac{d}{dx}\int_c^d f(x, y)\,dy = \int_c^d f_x(x, y)\,dy .$$

(b) Prove that $\int_0^{\pi/2} \log\left(a\cos^2\theta + b\sin^2\theta\right) d\theta = \pi \log \frac{\sqrt{a}+\sqrt{b}}{2}$.

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